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Solving Fuzzy Critical Path Problem Using Method of Magnitude

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Abstract: This paper deals with a novel method for solving critical path problems with fuzzy duration. The duration of activity is followed by normalized Trapezoidal fuzzy number. Method of magnitude is used for ranking of fuzzy numbers and for solving CPM.

Keywords: CPM, Magnitude of fuzzy number, method of promoter operator, parametric form of fuzzy number, Ranking of fuzzy numbers, Trapezoidal fuzzy number.

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1. Introduction

The critical path method (CPM) deals with all the ways of management science techniques developed in the late 1950s for planning, scheduling, and controlling large, complex projects with many activities. In the case of CPM, it is assumed that details about inputs are known with certainty, In this method a network representation is used to display the relationships between project activities and to help managers to address all the ways of questioning. An alternative way to deal with situations, involving imprecision in the data, is to employ the concept of fuzziness proposed by Zadeh (1965), whereby the vague activity times can be represented by fuzzy sets. The main objective of the optimized path problem is to find a path with minimum distance. The classical fuzzy shortest path problem introduced by Dubois and Prade [9]. They employed the fuzzy minimum operator to find the shortest path length, but they did not develop any method to decide the shortest path. They have used a fuzzy number instead of a real number assigned to each of the edges. Okada and Soper [22] concentrated on an optimized path problem and introduced the concept of degree of possibility in which an arc is on the optimized path. Another algorithm for this problem was presented by Okada and Klein [23,15] where there is a generalization of Dijkstra's algorithm. Various researchers have contributed many methods for solving Fuzzy CPM Problems.

A new approach namely defuzzification of fuzzy numbers with method of magnitude, that has been introduced to the generalization of the criticality notion for the case of the network with fuzzy activities duration times. In section 2 some elementary concepts and operations of fuzzy set theory have been reviewed. In section 3 method of magnitude, that has been introduced for ranking fuzzy numbers. In section 4, corresponding algorithms have been proposed for Fuzzy Shortest Path Problem. In section 5 the proposed method is illustrated by a numerical example taken from the exisisting method by liang and Hang (2004), P. Phani Bushan Rao and N. Ravi Shankar (2013) and some conclusions are drawn based on the results.

2. Preliminaries:

A fuzzy set **A** in **X** is characterized by a membership function $f_A(x)$ which associates with each point in **X** a real number in the interval **[0,1]**, with the values of $f_A(x)$ at x representing the "grade of membership" of x in **A**. Thus, the nearer the value of $f_A(x)$ to unity, the higher the grade of membership of x in **A**.

If the membership function $f_{\tilde{A}}(x)$ is piecewise linear, Then $\tilde{A}$ is said to be a trapezoidal fuzzy number. The membership function of a trapezoidal fuzzy number is given by

$$\mu_{\hat{a}(x)} = \begin{cases} w(x - a) / (b - a) & \text{if } a \le x \le b \\ w & \text{if } b \le x \le c \\ w(x - d) / (c - d) & \text{if } c \le x \le d \\ 0 & \text{otherwise} \end{cases}$$

If $w = 1$, then $\tilde{A} = (a, b, c, d; 1)$ is a normalized trapezoidal fuzzy number and $\tilde{A}$ is a generalized or non normal trapezoidal fuzzy number if $0 < w < 1$.The image of $\tilde{A} = (a, b, c, d; w)$ is given by $-\tilde{A} = (-d, -c, -b, -a; w)$.

In particular case if $b = c$, the trapezoidal fuzzy number reduces to a triangular fuzzy number given by $\tilde{A} = (a, b, d; w)$. The value of "b" corresponds with the mode or core and $[a, d]$ with the support. If $w = 1$, then $\tilde{A} = (a, b, d)$ is a normalized triangular fuzzy number $\tilde{A}$ is a generalized or non normal triangular fuzzy number if $0 < w < 1$.

Let $\breve{A} = (a_1, b_1, c_1, d_1)$ and $\breve{B} = (a_2, b_2, c_2, d_2)$ be two trapezoidal fuzzy numbers, then the fuzzy numbers addition and fuzzy numbers subtraction are defined as follows

(i)Fuzzy numbers addition of $\breve{A}$ and $\breve{B}$ is denoted by $\breve{A} \oplus \breve{B}$ and is given by

$\breve{A} \oplus \breve{B} = (a_1, b_1, c_1, d_1) \oplus (a_2, b_2, c_2, d_2) = (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2)$

(ii) Fuzzy numbers subtraction of $\breve{A}$ and $\breve{B}$ is denoted $\breve{A} \ominus \breve{B}$ and is given by

$\breve{A} \ominus \breve{B} = (a_1, b_1, c_1, d_1) \ominus (a_2, b_2, c_2, d_2) = (a_1 - d_2, b_1 - c_2, c_1 - b_2, d_1 - a_2)$

3 New approach for ranking of trapezoidal fuzzy numbers:

Method of magnitude [1]

For an arbitrary trapezoidal fuzzy number u = $(x_0, y_0, \sigma, \beta)$, with parametric form u = $(\overline{u}(r), \underline{u}(r))$, we define the magnitude of the trapezoidal fuzzy number u as

Mag(u)= $\frac{1}{2}\left(\int_0^1 \left(\overline{u}(r) + \underline{u}(r) + x_0 + y_0\right) f(r)dr\right)$,

where the function f(r) is a non-negative and increasing function on [0, 1] with f(0) = 0, f(1) = 1 and $\int_0^1 f(r)dr = 1$.

For example, we can use f(r) = r.

The resulting scalar value is used to rank the fuzzy numbers. In the other words Mag(u) is used to rank fuzzy numbers. The larger Mag(u), the larger fuzzy number.

Therefore for any two trapezoidal fuzzy numbers u and v ∈ E, we define the ranking of u and v by the magnitude on E as follows:

1. Mag(u) > Mag(v) if and only if u ≻ v,
2. Mag(u) < Mag(v) if and only if u≺v,
3. Mag(u) = Mag(v) if and only if u ≈ v.

Then we formulate the order ≽and ≼ as u ≽ v if and only if u ≻ v or u ≈ v, u ≼ v if and only if u ≺ v or u ≈ v. In the Other words, this method is placed in the first class of Kerre's Categories .

Method of promoter operator [2]

Let u = (a, b, c, d) be a non-normal trapezoidal fuzzy numbers with *r*-cut representation u = ($\overline{u}$(r), $\underline{u}$(r)), consequently we have

$$\text{Mag (u)} = \frac{(3w^2+2)(b+c)}{12w}, \frac{(3w-2)(a+d)}{12w}$$

It is clear that for normal trapezoidal fuzzy numbers the above formula can be reduced to

$$\text{Mag(u)} = \frac{5}{12}(b+c) + \frac{1}{12}(a+d)$$

4. The CPM with crisp activity times

A fuzzy project network S = < V, A, t > being a project model, is given. V is a set of nodes (events) and $A \subset V \times V$ is a set of arcs (activities). The network S is a finite, directed, compact, acyclic graph. The set V ={1, 2,….n}is labeled in such a way that the following condition holds: (i , j) ∈ A ⇒i< j.

For each activity a_{ij}, a fuzzy number $t_{ij} \in T$ is defined, where t_{ij} is the fuzzy time required for the completion of a_{ij}

By means of function t; t: A→R+, the activity times in the network are determined, t (i, j) $\overset{def}{\Longrightarrow}$ t_{ij} is a duration of activity (i, j) ∈A.

The essence of the CPM method (from numerical point of view) are two recurrence formulae which are used to determine the earliest and the latest moments of occurring the events i ∈V.

Let us denote by P(i)={k ∈V | (k ,i)∈A} the set of predecessors and by S(i) ={k ∈V | (i, k))∈A} the set of successors of event i ∈V, respectively. The earliest moments, T_i^e of occurrence of the events i ∈V are determined by means of the following recurrence formula:

$$T_i^e = \begin{cases} 0 & for\ i = 1 \\ max_{k \in p(i)(T_k^e + t_{ki})} & for\ i > 1 \end{cases}$$

And the latest moments, $T_i^l$of occurrence of the events i ∈V can be found by use of the following formula:

$$T_i^l = \begin{cases} T_n^e & for\ i = n \\ min_{k \in s(i)}\left(T_k^l - t_{ik}\right) & for\ i < n \end{cases}$$

The times obtained by the use of the above formulas are applied to the calculation of slack times, $L_i = T_i^l - T_i^e$,for events i ∈V and slack times, Z(i,j) = $T_j^l - T_i^e - t_{ij}$ for activities (i, j)∈A. By means of such determined quantities the notions of criticality of an event and of an activity can be defined.

Definition 1. An activity (i, j) ∈A is critical if and only if Z (i, j) =0.

Definition 2. An event i ∈V is critical if and only if L_i =0.

Let us denote by P the set of all paths in S from node 1 to node n.

Definition 3. A path p ∈P is critical if and only if all activities belonging to *p* are critical.

The following theorems are obvious.

Theorem 1. A path p ∈P is critical if and only if it is the longest path in the network S with the lengths of arcs equal to t_{ij} , (i , j)∈A. The length of this path is equal to T_n^e .

Theorem 2. An activity (i, j) ∈A (an event i ∈ V) is critical if and only if it belongs to a certain critical path p ∈ P.

The problems of determining critical activities, events and paths are easy ones in a network with deterministic (crisp) durations of activities. If we remove from the network all the non-critical activities then we obtain the network in which all the paths leading from the initial node 1 to the end node n are critical ones (with the same length equal to T_n^e). Naturally, the number of these paths may be very great as it increases exponentially together with the network size extension. However, the network reduced to the set of critical activities is available in a time bounded by a polynomial in the size of the initial network.

Notations

N = {1, 2, 3... n}: the set of all the nodes in a project network.

A_{ij} : the activity between nodes *i* and *j*

$F\hat{A}T_{ij}^n$: the fuzzy normal time of activity A_{ij}

$F\hat{E}S_j$: the fuzzy earliest starting time of node *j*

$F\hat{L}F_j$: the fuzzy latest finishing time of node *j*

$F\widehat{T}F_{ij}$: the fuzzy total floats time of activity A_{ij}

S (j) : the set of all successor activities of node *j*

F (j) : the set of all predecessor activities of node *j*

NS (j) : the set of all nodes connected to all successor activities of node *j*, i.e.

NS(j) = { k / $A_{jk} \in S(j)$, K∈ *N* }

N*P (j)* : The set of all nodes connected to all predecessor activities of node *j*, i.e

NP(j) = { i / $A_{ij} \in F(j)$, i∈ *N* }

P_i : the *i-th* path

P : the set of all paths in a project network

$FC\hat{T}$ (P_c) : the fuzzy completion time of the fuzzy critical path *PC* in a project network.

Properties

Outlines of the important properties that will be used in the proposed method and to solve the numerical examples as follows:

Set the initial node as zero for starting i.e. $F\hat{E}S_1$ = (0, 0, 0, 0) then the following will hold.

1. $F\hat{E}S_i = max\ \{F\hat{E}S_i \oplus F\hat{A}\,T_{ij}^n / \ i \in NP(j), j \neq 1, j \in N\}$

2. $F\hat{L}F_j = \min \{ F\hat{L}S_i \ominus F\hat{A}\,T_{ij}^n / \ k \in NP(j), j \neq n, j \in N$

3. $F\hat{T}F_{ij} = (F\hat{L}F_j \ominus F\hat{E}S_i) \ominus F\hat{A}\,T_{ij}^n, 1 \le i < j \le n; j \in N$

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4.$FC\widehat{T}\,(P_C)$=$\sum_{i,j\in p_c} F\hat{A}\,T_{ij}^{n}, P_c \in P$

Definition: Assume that there exists an activity A_{ij} in a project network with the property min { $F\widehat{T}F_{ij}$, ∀ A_{ij} }, then the activity A_{ij} is said to be a critical activity.

Proposed method:

For finding the path of a project network in a fuzzy environment a fuzzy critical path analysis algorithm is developed. A project network is constructed after identifying the activities in a project and establishing the precedence relationships of all activities along with the fuzzy normal time with respect to each activity,

The algorithm of the proposed method

Step 1: Set $F\hat{E}S_1$ = (0, 0, 0, 0) and calculate $F\hat{E}S_j$, j = 2,3,....,n by using the property of 1

Step 2: Set $F\hat{L}F_n = F\hat{E}S_n$ and calculate $F\hat{L}F_j$, j = n−1, n − 2,...,1 by using property 2

Step 3: Calculate $F\widehat{T}F_{ij}$ with respect to each activity in a project network by using property 3 .

Step 4: Find fuzzy critical path of the network by definition 8.

Step 5: Find fuzzy critical path P_c by combining all the fuzzy critical activities obtained in step 4.

Step 6: Calculate $FC\widehat{T}$ (P_c) of the fuzzy critical path P_c of a project network obtained in step 5 by using property 4,and find the project completion time .

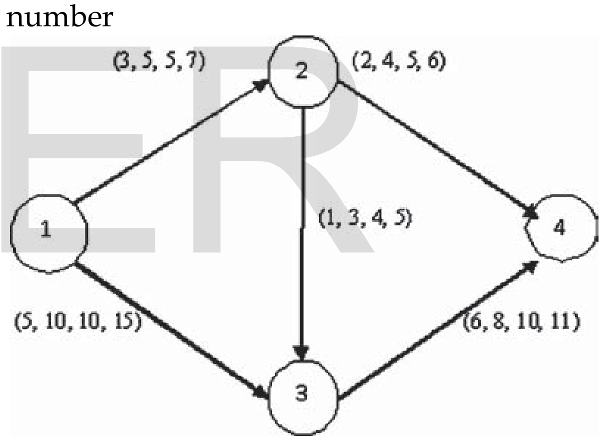
5 Numerical example

In this section, a hypothetical project problem is presented to demonstrate the computational process of the fuzzy critical path analysis that has been proposed. The proposed method is illustrated by solving the numerical example chosen from the literature (Liang andHan,2004),and it is shown that the result in the existing method are as same as the proposed method. Here the method of magnitude is used for ranking the fuzzy numbers.

Example: Suppose there is a project network, as shown in Figure 2, with the set of nodes N = {1, 2, 3, 4}, and the fuzzy activity time for each activity is taken as a trapezoidal fuzzy number.

The problem is to find the fuzzy earliest starting time, fuzzy latest finishing time, fuzzy critical path and fuzzy completion time of the project network shown in Figure 2, in which the fuzzy normal time of each activity is represented by the following trapezoidal numbers.

Project network with fuzzy time duration of each activity as trapezoidal fuzzy number



FAT_{12}^{n}= (3,5,5,7), FAT_{13}^{n} = (5,10,10,15),
FAT_{23}^{n} = (1,3,4,5),
FAT_{24}^{n}= (2,4,5,6), FAT_{34}^{n} = (6,8,10,11)

Solution:

The fuzzy earliest starting time, fuzzy latest finishing time, fuzzy critical path and fuzzy completion time of the project network shown in Figure 2 can be obtained by using the following steps:

Step 1: By assuming $F\hat{E}S_1$ = (0, 0, 0, 0) the values of $F\hat{E}S_j$, j = 2, 3, 4 can be obtained as follows:

$F\hat{E}S_2 = F\hat{E}S_1 \oplus FAT\,{}^{n}_{12}$ = (0,0,0,0) ⊕ (3,5,5,7) = (3,5,5,7)

$F\hat{E}S_3 = max\,\{F\hat{E}S_1 \oplus FAT\,{}^{n}_{13}, F\hat{E}S_2 \oplus FAT\,{}^{n}_{23}\}$

= max{ (0,0,0,0) ⊕ (5,10,101,15),(3,5,5,7) ⊕ (1,3,4,5)}

= max{ (5,10,101,15), (4,8,9,12)}

$\Re(5,10,10,15) = \frac{5}{12}(10+10) + \frac{1}{12}(5+15) = 9.9$

$\Re(4,8,9,12) = \frac{5}{12}(8+9) + \frac{1}{12}(4+12) = 8.4$

Since ℜ(5,10,10,15) > ℜ(4,8,9,12)

i.e., $F\hat{E}S_3$ = (5, 10, 10, 15)

$F\hat{E}S_4 = max\{F\hat{E}S_3 \oplus FAT\,{}^{n}_{34}, F\hat{E}S_2 \oplus FAT\,{}^{n}_{24}\}$

= max{(5,10,10,15) ⊕ (6,8,10,11),(3,5,5,7) ⊕ (2, 4,5,6)}

= max{(11,18, 20, 26),(5,9,10,13)}

ℜ(11,18, 20, 26) = 18.9, ℜ(5,9,10,13) = 9.4

Since ℜ(11,18, 20, 26) > ℜ(5,9,10,13)

So, maximum{(11,18, 20,26),(5,9,10,13)} = (11,18, 20, 26)

i.e. $F\hat{E}S_4$ = (11 18 20 26)

Step 2: By assuming $F\hat{L}F_4$ = (11 18 20 26) the values of $F\hat{L}F_3$, j =3 ,2,1 can be obtained as follows:

$F\hat{L}F_3 = F\hat{L}F_4 \ominus FAT\,{}^{n}_{34}$ = (11,18,20,26) ⊖ (6,8,10,11) = (0,8,12,20)

$F\hat{L}F_2 = \min\{F\hat{L}F_4 \ominus FAT\,{}^{n}_{24}, F\hat{L}F_3 \ominus FAT\,{}^{n}_{23}\}$

= min {(11, 18, 20, 26) ⊖ (2, 4, 5, 6), (0, 8, 12, 20) ⊖ (1, 3, 4, 5)}

= min {(5, 13, 16, 24), (–5, 4, 9, 19)}

ℜ(5,13,16, 24) = 14.5, ℜ(−5, 4,9,19) = 6.5

Since ℜ(5,13,16,24) > ℜ(−5,4,9,19)

So, minimum {(5, 13, 16, 24), (–5, 4, 9, 19)} = (–5, 4, 9, 19)

i.e., $F\hat{L}F_2$ = (–5, 4, 9, 19)

$F\hat{L}F_1 = \min\{F\hat{L}F_3 \ominus FAT\,{}^{n}_{24}, F\hat{L}F_3 \ominus FAT\,{}^{n}_{23}\}$

= min {(0, 8, 12, 20) ⊖_(5, 10, 10, 15), (–5, 4, 9, 19) ⊖ _(3, 5, 5, 7)}

= min {(–15, –2, 2, 15), (–12, –1, 4, 16)}

ℜ (−15, −2,2,15) = 0, ℜ (−12, −1,4,16) = 1.58

Since ℜ (−15, −2,2,15) < ℜ (−12, −1,4,16)

So, minimum {(–15, –2, 2, 15), (–12, –1, 4, 16)} = (–15, –2, 2, 15)

i.e., $F\hat{L}F_1$ = (–15, –2, 2, 15)

***Step 3*:** Calculate $F\hat{T}F_{ij}$ with respect to each activity by using property 3

$F\hat{T}F_{12} = (F\hat{L}F_2 \ominus F\hat{E}S_1) \ominus F\hat{A}\,T^{n}_{12}$

= ((–5, 4, 9, 19) ⊖ (0, 0, 0, 0)) ⊖ (3, 5, 5, 7)

= (–12, –1, 4, 16)

$F\hat{T}F_{13} = (F\hat{L}F_3 \ominus F\hat{E}S_1) \ominus F\hat{A}\,T^{n}_{13}$

= ((0, 8, 12, 20) _(0, 0, 0, 0)) _(5, 10, 10, 15)

= (–15, –2, 2, 15)

$F\hat{T}F_{23} = (F\hat{L}F_3 \ominus F\hat{E}S_2) \ominus F\hat{A}\,T^{n}_{23}$

= ((0, 8, 12, 20) ⊖ (3, 5, 5, 7)) ⊖ (1, 3, 4, 5)

= (–12, –1, 4, 16)

$F\hat{T}F_{24} = (F\hat{L}F_4 \ominus F\hat{E}S_2) \ominus F\hat{A}\,T^{n}_{24}$

= ((11, 18, 20, 26) ⊖ (3, 5, 5, 7)) ⊖(2, 4, 5, 6)

= (–2, 8, 11, 21)

$F\hat{T}F_{34} = (F\hat{L}F_4 \ominus F\hat{E}S_3) \ominus F\hat{A}\,T^{n}_{34}$

= ((11, 18, 20, 26) ⊖ (5, 10, 10, 15)) ⊖ (6, 8, 10, 11)

= (–15, –2, 2, 15)

Step 4: Find the fuzzy critical activity of a project by definition 8.

ℜ ($F\hat{T}F_{12}$) = ℜ (−12 , −1 ,4 ,16) = 1.58

ℜ($F\hat{L}F_{13}$) = ℜ(−15 , −2 ,2 , 15) = 0

ℜ ($F\hat{T}F_{23}$) = ℜ (−12 , −1 ,4 ,16) = 1.58

ℜ($F\hat{T}F_{24}$) = ℜ(−2 ,8 ,11 ,21) = 9.5

ℜ($F\hat{T}F_{34}$) = ℜ(−15 , − 2,2 ,15) = 0

Hence, activities (1, 3) and (3, 4) are fuzzy critical activities.

Step 5: Combining all the fuzzy critical activities obtained in step 4, the fuzzy critical path is1⇒3⇒ 4(*say* P_c).

Step 6: Calculate $FC\hat{T}$ (P_c) of the fuzzy critical path Pc of a project network obtained in step 5 by using property 4:

$FC\hat{T}$ (P_c) = $(FAT\ ^{n}_{13} \oplus FAT\ ^{n}_{34})$ = (5 10 10 15) ⊕ (6 8 10 11) = (11 18 20, 26)

The project completion time is approximately between 18 and 20 hours(11, 18, 20, 26).Comparing the results with the same example by liang and Hang (2004), P. Phani Bushan Rao and N. Ravi Shankar (2013) by using the method of magnitude the results are given in tabular form

P. Phani Bushan Rao and N. Ravi Shankar (2013)	**Proposed method of magnitude**
$\Re$(5,10,10,15) = 10	$\Re$(5,10,10,15) =9.9
$\Re$(4,8,9,12) = 8. 38	$\Re$(4,8,9,12) =8.4
$\Re$(11,18, 20, 26) = 18.88	$\Re$(11,18, 20, 26) = 18.9
$\Re$(5,9,10,13) = 9.38	$\Re$(5,9,10,13) = 9.4
$\Re$(5,13,16, 24) = 14.5 $\Re$(−5, 4,9,19) = 6.61	$\Re$(5,13,16, 24) = 14.5 $\Re$(−5, 4,9,19) = 6.5
$\Re$ (−15, −2,2,15) = 0, $\Re$ (−12, −1,4,16) = 1.61	$\Re$ (−15, −2,2,15) = 0, $\Re$ (−12, −1,4,16) = 1.58
$\Re$ ($F\hat{T}F_{12}$) = $\Re$ (−12 , −1 ,4 ,16) = 1 .61	$\Re$ ($F\hat{T}F_{12}$) = $\Re$ (−12, −1, 4, 16) = 1. 58
$\Re$($F\hat{L}F_{13}$) = $\Re$(−15 , −2 ,2 , 15) = 0	$\Re$($F\hat{L}F_{13}$) = $\Re$(−15 , −2 ,2 , 15) = 0
$\Re$ ($F\hat{T}F_{23}$) = $\Re$ (−12 , −1 ,4 ,16) = 1.61	$\Re$ ($F\hat{T}F_{23}$) = $\Re$ (−12 , −1 ,4 ,16) = 1.58
$\Re$($F\hat{T}F_{24}$) = $\Re$(−2 ,8 ,11 ,21) = 9.5	$\Re$($F\hat{T}F_{24}$) = $\Re$(−2 ,8 ,11 ,21) = 9.5
$\Re$($F\hat{T}F_{34}$) = $\Re$(−15 , −2,2 ,15) = 0	$\Re$($F\hat{T}F_{34}$) = $\Re$(−15 , −2,2 ,15) = 0

By comparing the above shortcomings we get a better approximation.

6. Conclusions

The proposed method can effectively rank various fuzzy numbers. The proposed method can be used to find the critical path of a project network that exists in real life situation. CPM is commonly used with all forms of projects, including construction, aerospace and defense, software development, research projects, product development, engineering, and plant maintenance. Fuzzy ranking techniques can be implemented in the above forms of projects for getting better approximation. The comparison reveal that the method proposed in this paper has shown more effective in determining the activity criticalities and finding the critical path.

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